Exploiting Semantic Indexing Images for Emergence Recommendation Semantics System

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Abstract – Thanks to the efforts of the Semantic Web Community (W3C), images can be semantically indexed with metadata. The explicit representation of image contents is made possible by using ontologies that provide a common and shared understanding of a domain at both human users and application levels. The approach that we are proposing in this paper is a semantic indexing of images based on conceptual method. To make efficient the semantic indexing, we also propose a recommender system. User profiles: static and dynamic profiles are combined and supported by the system we have developed to suggest recommendations to users. The preferences of each user are taking into account to provide customized recommendations.

Keywords- image; semantics; ontology; indexing; recommendation

I. INTRODUCTION

The growth of multimedia data and in particular images caused not only a need for storage but also the need to get access to those images. So as to these data to be usable, they need to be effectively referred back to as in a catalogue. The techniques presented below, called indexing, propose to attach to an image a set of descriptors that describe their content.

Many approaches seek the use of semantics to extract the representative of images content descriptors. These descriptors are then used to allow the system to retrieve the images of interest to the user. A set of keywords, names, nominal sentences are mapped to the concepts that they represent [2]. In these approaches, an image is represented as a set of concepts. To achieve this, the semantic structures of image representations are needed. These structures can be dictionaries, taxonomies or ontologies [3]. They can be either manually or automatically generated. They are widely used to improve the efficiency of images retrieval. There are generally three types of indexing: classic, conceptual and ontological. The classical indexing is based either on lexical or syntaxical analysis of the images content by taking into account keywords occurrences. The conceptual indexing is a statistical approach that aims to extract the semantics contained in the images. This approach groups terms that have common features in images and considers that each group represents a semantic. The terms chosen should allow to

retrieve the relevant images with respect to the representation of user needs. Two parameters are taken into account in classical indexing: language of representation and discriminating power. [4][5][6] [7].

A new generation of methods is to consider the concepts rather than words. The conceptual indexing allows to identify the concepts and / or instances of ontology that appear in the images. The approach proposed in [15] aims to understand the specific requirements of users in order to meet their needs. Users propose instances of concept in their queries such as the acquisition time and the type of sensor and especially select the concepts by which they are interested. The researchers also assumed that the terms can carry a semantic structure whose they try to extract the concepts as a unit of semantic. To achieve this purpose, several approaches have emerged. The approach proposed in [7][8][9] aims to avoid the polysemy and synonymy of terms used as descriptors by conventional statistical approaches. It groups terms with common characteristics in their appearance in the images. Another approach consists of identify the elements of the ontology. This approach was used in [10] [11][12][13]. Other approaches extract "expressions". The extraction of expressions is important because the instances of the concepts are often composed of such elements [10][14].

Another type of ontological indexing approach is to rely on ontologies to retrieve images; this type of indexing is called ontological indexing. The ontological indexing put forward the fact that the meaning of textual information depends on the conceptual relationships between the objects to which they refer to [1] [16]. Ontological indexing is possible only by the existence and use of resources explicitly describing the information corresponding to objects [17][18]. Regarding ontology usage for images indexing, Khan [19] proposed a method based on sub-trees "regions" of ontology. Regions of an ontology represent different concepts. Concepts that appear in a given region are mutually disjoint concepts from other regions. The region containing the largest number of concepts is selected. Then all the selected concepts that also appear in other regions are deleted. In a region, the selection is made through the use of "semantic distances", by taking into account of paths between

concepts in the ontology. The concepts that correlate with the greatest number of other concepts are selected for indexing. Woods [20] proposed the same method of indexing, but his approach retrieves similar images.

The studies performed in this paper consist in comparing the existing semantics indexing methods through large collections of images and present their advantages. Following, we will propose a technique that meets better the needs of users. Finally, we will propose a recommendation system for the semantics used by users to retrieve images. This includes a new factor to better take into account the user interactions with the retrieval system to perform specific recommendations.

II. EMERGSEM APPROACH

We note that once the images are annotated, different needs for access to these images appeared, each corresponding to a specific action: sequential scan of a set of images in the case where the user does not really have idea of what he wants, image search, when the user knows exactly what he wants and finally the image classification, which helps users to combine images with similar features and thus provides a simplified representation of an ordered set of images.

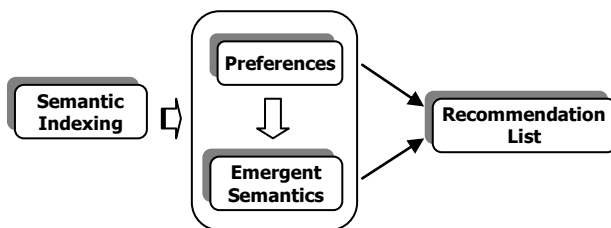The architecture of the suggested system is shown in Figure 1.

Figure 1. EMERGSEM model

We propose the next two steps to realize EMERGSEM system. The system is equipped with image retrieval functionality and recommendations ranking technique to facilitate image retrieval and recommendations generation. The approach proposes a technique based on ontology concepts. Concepts are used to select image semantics that are then used to retrieve images. Then, the system makes semantics recommendation using three fundamental steps: acquiring preferences from the user's input data, computing recommendations and suggesting the recommendation to the user.

III. SEMANTIC INDEXING

In this section, we present an approach for semantic indexing of images based on concepts. To facilitate semantic indexing of images, we propose a conceptual indexing to end users. So users can provide keywords that represent instances of concepts of ontologies used to store the semantics of images they want as we show in Figure 2.
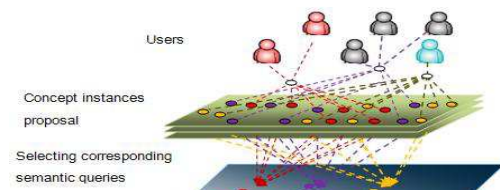

Figure 2. Semantic indexing

The system selects the most similar semantics corresponding to instances proposed by user i.e., the concepts belonging to semantic of the retrieved image. This selection is based on the following algorithm.

Figure 3. Retrieval Algorithm

For k_i instances submitted by the user, we define the probability that these instances appear in the instances of each semantic displayed N_c. The retrieval probability is then computed as follows:

$$\text{Prob} = \frac{\sum k_i}{N_c} \quad (1)$$

Based on the semantic interpretations provided, the similar semantics containing these instances can be obtained. Lastly, the image semantics that contain greater instances are retrieved. Subsequently, the user can select the appropriate semantic of semantics displayed to index the image.

The process of the concept-based image retrieval depicted can be described as follows:

(1) User proposes keywords indicating the content of images.

(2) EMERGSEM system verifies the presence of the instances proposed by user in each concept of the ontologies.

(3) Once the instances are validated, the program invokes an ontology query service. The semantics containing the instances are displayed thanks to the ontology search engine.

(4) User selects the semantic that describes better the needs image. Image is retrieved from the image database.

(5) Finally, the results are displayed to the user.

IV. RECOMMENDATION SYSTEM

Recommendation system has been a hot research topic in recent years. To recommend items to a user, the system must have a representative profile preference. To build this, it must collect

information about it, either directly or indirectly [21][22]. Recommender systems help users to manage information overload by providing personalized advice on content and online services. The term "recommendation system" generally describes a system that produces customized recommendations to users, and has the effect of leading the user to interesting items in a large space of possible options [23][25].

A recommendation system we propose aims to recommend images semantics to a user in correspondence with its tastes and preferences. The aim is both to minimize the time spent on research, but also to suggest relevant semantics that would not be spontaneously consulted and increase the overall satisfaction of users.

The first step in the realization of the recommendation system is to extract the profiles of users. The next section focuses on this purpose.

A. Acquiring Profiles

Acquisition of user profiles is composed of two important steps: extraction of static and dynamic profiles.

The static profiles are also called independent part of the domain. They take into account any data that has no connection with the domain. There may be personal user information such as professional status. This part does not require large resources since users, before using the system are required to create an account and thus to provide such personal information.

The dynamic profiles are also called dependent part of the domain or the active model. It consists of data that represent the needs, interests and goals of the user i.e., user preferences. This part will be constructed by the system in response to user interactions with the system: a history of the user's interactions with the recommendation system. To achieve this, the system needs to collect such data on assessments of the user. The analysis of these data is then used to build a model of the user's preferences that will be used by the system to recommend the semantics deemed relevant for the user. A model of the user's preferences, i.e., a description of the types of semantic that interest the user is represented. There are many possible alternative representations of this description, but one common representation is a function that for any semantic predicts the likelihood that the user is interested in that semantic. For efficiency purposes, this function may be used to retrieve the n semantics most likely to be of interest to the user [26].

B. Classification of Preferences

What we learned from this work is that the comparison between the profiles leads to the formation of user groups close to each other, groups called "communities". So we can say that the notion of community is a key factor in a recommender system to produce recommendations [24]. It is clear

that the positioning of users in the spaces depends crucially on the dynamic profiles. The dynamic profiles of each user then evolve along with the user himself.

To group profiles, we think that the formal concept analysis [29][30][31] and Galois lattices [32][33][34] will be indispensable. A lattice of Galois can regroup, exhaustively objects in classes, called "formal concept", using their shared properties. A lattice is typically based on a Boolean matrix, called matrix context and denoted C, whose rows represent a set of objects O that we wish to describe and columns, a set of attributes A that these objects have or have not.

TABLE I. FORMAL CONCEPT OF SEMANTICS

Dynamic Profiles	Static Profiles			
	User 1	User 2	User 3	User 4
Semantic 1	X			X
Semantic 2		X	X	
Semantic 3	X	X	X	
Semantic 4				X
Semantic 5			X	
Semantic 6		X		
Semantic 7		X		X

Suppose we have a description of the following profiles (see Table I). This description is based on the list of dynamic profiles that users have chosen or not. Possession of the property $a \in A$ by object $o \in O$ reflects the existence of a relationship I between them: aIo. The existence of this relation I between O and A is materialized in the matrix of context C by a value "true" or "false". The triplet $K = (O, A, I)$ is called a formal context or context. A set $X \subset O$ is the set of attributes jointly owned by all object X and is given by the function f:

$$f(X) = \{a \in A | \forall o \in X, oIa\} \quad (2)$$

Inversely a set $Y \subset A$ is the set of objects jointly owned by all object Y and is given by the function g: $g(Y) = \{o \in O | \forall a \in Y, oIa\}$ (3)

The pair (f, g) is called a Galois connection. A concept is any pair $C = (X, Y) \subseteq O \times A$, such that objects in X are the only one to have attributes in Y; in other words X x Y is, if we add permutations of O and A, a maximal rectangle in C, i.e.

$$f(X) = Y \& g(Y) = X. \quad (4)$$

To illustrate this approach of formal Concept, the Table I shows that the set X= {Semantic 2, Semantic 3} gives one formal concept since $f(X) = \{User 2, User 3\} = Y$ et $g(Y) = X$, and this formal concept is ({Semantic 2, Semantic 3}, {User 2, User 3}), while the set X' = {Semantic 1, Semantic 4} doesn't give a formal concept because $f(X') = \{User 4\} = Y'$ et $g(Y') = \{Semantic 1, Semantic 4, Semantic 7\}$

The Galois lattice is represented by a Hasse diagram as shown in figures 4. "Sem" means Semantic.
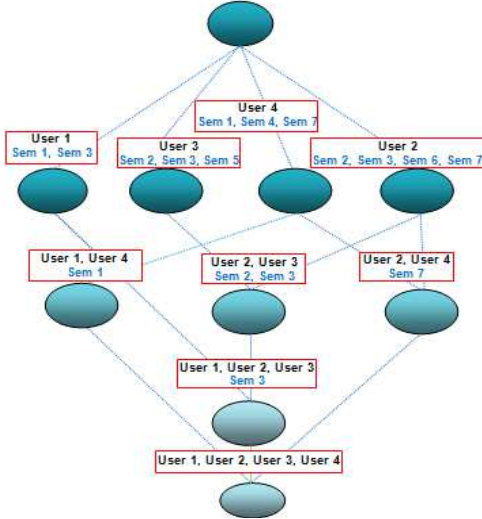
Figure 4: Display Galois lattice of users profiles

In this Figure, the static profiles are connected to dynamic profiles. For example we see that the users 1, 2 and 3 have selected semantics 3 while the users 2 and 4 chose the semantic 7.

One of important advantages of classification based on Galois lattice is that for a given formal context table the resulting lattice is unique, and it is exhaustive. This classification will allow us to find all the groups of static profiles in relation with a group of dynamic profiles and represent them similarly.

C. Emergent Semantic

Image semantics are provided to users after the proposition of instances. Users choose appropriate semantic in the list, i.e., the semantic that better meet their research needs. Once a semantic is used to search for images, a weight is assigned by the recommendation system which is responsible for the link between the semantic and the user. The users interact with the system, and the semantics used will be evaluated by the system. The system can therefore recommend these semantic to them. The weight is calculated by:

$$Wht_{i,k} = \sum Pbt_{i,k}, (5)$$

where Pbt is the probability of a semantic i to be chosen in image k.

D. Recommendation List

Recommendation list may then be introduced, once user profiles are grouped. It is to look for similarities between the dynamic profiles of each constituted group to make customized recommendations. Similarity measures considered here satisfy the following properties for all (u, v) ∈ U:

- sim (u, v) ∈ [0; 1]; (sim mean similarity)
- sim (u, v) = 1; if and only if u and v have the same common profile;
- sim (u, v) = 0; If u and v do not have a elements of comparison [24].

The method chosen to determine the user's profiles similarity is cosine similarity since the cosine similarity seems very promising. It provides an accurate measure of similarity [27][28].

The recommendation list given to user is consisted of two parameters: the personalized recommendations representing the preferences of each user and the general recommendations representing a mostly used semantic of each image, that are the emergent semantics. Unlike specific recommendations the emergent semantic is recommended to all users.

Let u_1 and u_2 be two users with dynamic profiles specifying their utility functions of the subsets $I_1 \subseteq I$ and $I_2 \subseteq I$. We then calculate the similarity typically using for example the cosine similarity [25] by:

$$cos_{ut}(ut_{u_1}, ut_{u_2}) = \frac{\sum_{i \in I_1 \cap I_2} ut_{u_1}(i) \cdot ut_{u_2}(i)}{\sqrt{\sum_{i \in I_1} ut_{u_1}(i)^2} \cdot \sqrt{\sum_{i \in I_2} ut_{u_2}(i)^2}} \quad (6)$$

If $cos_{ut}(ut_{u1}, ut_{u2}) \geq$ Threshold then u_1 and u_2 are similar. Note that the threshold varies from one image to another, and this threshold is not stable. In order to compute the threshold between the instances of concepts, we calculate the average of common with proposed instances. Let x and y denote the feature sets of the common and proposed instances, respectively for image I. The threshold is,

$$Threshold = \frac{\sum x_I}{\sum y_I}, (7)$$

For example for image 1 (Table III, Table IV), the threshold= 15/21= 0,714.

V. EXPERIMENTATION

To illustrate how the combined approaches perform in practice, it was evaluated on a real-world semantics recommendation application and compared its performance with the simple semantics indexing. The following table (Table II) gives the variations of images indexing durations. We note that the data in the first column of the table (T_i (hour)) are higher than those of other columns which remain stable.

The fundamental reason is that the first index is performed without any semantic recommendation. Users have suggested instances of concepts to retrieve images. But in the other columns, the search time is greatly reduced because the system took into account the preferences of the users to make their recommendation. The result is that they have a huge time saver. The following figure (Figure 5) shows indexing duration variation.

TABLE II. EXAMPLE OF INDEXING DURATION

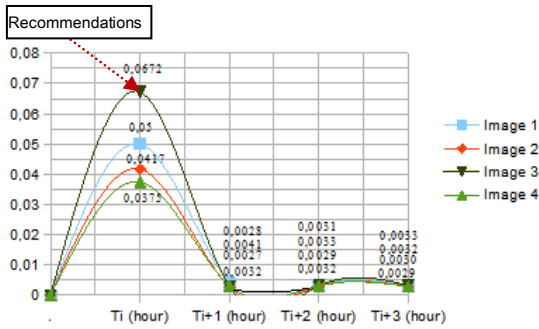|	T_i (hour)	T_{i+1} (hour)	T_{i+2} (hour)	T_{i+3} (hour)
Image 1	0,0500	0,0041	0,0033	0,0032
Image 2	0,0417	0,0027	0,0029	0,0030
Image 3	0,0672	0,0028	0,0031	0,0033
Image 4	0,0375	0,0032	0,0032	0,0029

Figure 5: Indexing duration

The figure 5 shows a graph with four curves representing the indexing duration of the 4 images. We note that the curves decrease when users had the recommended semantics for images semantic indexing. Then, the curves remained stable with values around 10.8 seconds. The recommendation system has helped to save time during semantic indexing.

The following tables (Table III, IV and V) shows the different steps of calculating the similarity between user profiles to make them recommendations based on their preferences. Table 3 is an example of instances of concepts proposed by users.

TABLE III. CALCULATION OF INSTANCES PROPOSED BY SEMANTIC
U means User, and Sem means Semantic

	Instances proposed								
	U_1			U_2			U_3		
	Sem 1	Sem 2	Sem 3	Sem 1	Sem 2	Sem 3	Sem 1	Sem 2	Sem 3
Image 1	08	-	-	-	07	-	06	-	-
Image 2	-	06	-	10	-	-	-	07	-
Image 3	-	-	08	-	09	11	-	-	
Image 4	05	-	-	-	04	04	-	-	-

TABLE IV. IDENTICAL INSTANCES OF CONCEPTS BETWEEN USERS

	$U_i, U_{j(i \neq j)}$		
	$U_1 \cap U_2$	$U_1 \cap U_3$	$U_2 \cap U_3$
Image 1	04	07	04
Image 2	05	02	08
Image 3	07	04	06
Image 4	02	02	04

TABLE V. DYNAMICS PROFILES SIMILARITY BASED ON COSINE

	$cos_{ut}(ut_{u_1}, ut_{u_2})$		
	$U_1 \cap U_2$	$U_1 \cap U_3$	$U_2 \cap U_3$
Image 1	0,2539	0,8750	0,3809
Image 2	0,4167	0,0952	0,9142
Image 3	0,6805	0,1818	0,3636
Image 4	0,2000	0,1142	0,5714

We note that for the image 1, user 1 proposes 8 instances of semantic 1 while user 2 provides 7 instances of semantic 2 and user 3 has 6 instances of semantic 1, etc. Common instances of each user are listed in Table IV (example: Among the proposals made by users 1 and 2 in Table III, IV instances are identical). Table V gives information

on the results of the similarities between the dynamic profiles. To make recommendations of a semantic to a user group, only dynamic profiles that have a similarity greater or equal to the threshold are recommended for users concerned (see Figure 6), i.e., the profiles are considered similar according to the cosine similarity.
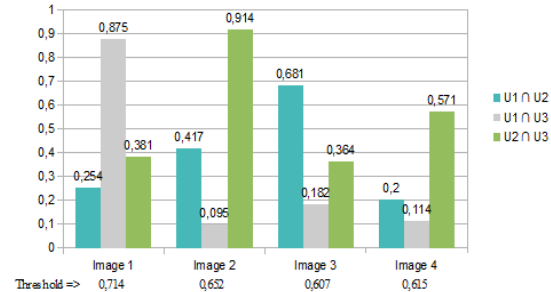

Figure 6: Similar dynamic profiles determination

The semantics of the image 1 can be recommended to users 1 and 3, the semantics of image 3 will be suggested to user 1 and 2, and those of image 2 to the users 2 and 3.

We compare our method with two classifications methods (SVM and Naive Bayes). The next table shows the result of our experiment. Three parameters are taken into account: the performance (possibility to reduce the errors) and classification time.

TABLE VI. COMPARING OF CLASSIFICATION APPROACHES

	Performance (Error Reduction)	(Classification time) (minutes))
Naive Bayes	89,72%	14,03
SVM	81,07%	09,62
Galois	91,18%	09,18

The experiment was conducted on 632 profiles of users on different classifiers. The results are presented in the Table VI. We note that all methods are efficient because they reduce significantly the error rate with different classification time. We find that the results of our approach are better because it can regroup, exhaustively objects in classes. Although there is a tradeoff between complexity and performance, it is still viable choices when better performance is considered.

VI.   CONCLUSION

In this paper, we propose an efficient method for semantics recommendation based on indexing. We first formalize semantic indexing based on concepts of ontology. Then, we propose a similarity by exploiting the relationship between dynamic profiles. The similarities are used to make tighter recommendation of semantics to the users. The purpose of this recommendation system can be achieved through the management of static and dynamic profiles derived from semantic indexing.

The combination of semantic indexing and recommendation system calls for the development of more flexible recommendation methods that

allow the user to express the types of recommendations that are of interest to them rather than being "hard-wired" into the recommendation engines provided by most of the current vendors that, primarily, focus on recommending semantics to the user and vice versa. The second requirement of interactivity also calls for the development of tools allowing users to provide inputs into the recommendation process in an interactive and iterative manner, preferably via some well-defined user interface.

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