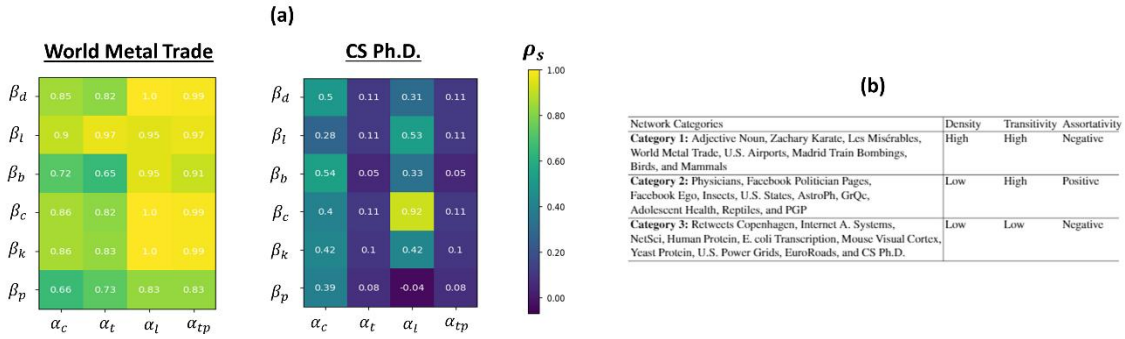
Hierarchy and Centrality: Two Sides of The Same Coin?

S. Rajeh[1], *M. Savonnet*[1], *E. Leclercq*[1], *H. Cherifi*[1]. *Laboratoire D'Informatique de Bourgogne, University of Burgundy, 21000 - Dijon, France, stephany.rajeh@u-bourgogne.fr.*

Identifying influential nodes that can spread information throughout a network, suppress disease outbreaks, or fail terrorist attacks is a fundamental issue. Centrality and hierarchy measures come forth in quantifying node influence. The former leverages dynamics while the latter exploits the hierarchical structure of networks. Several works have been devoted to centrality measures, however, the relationship between hierarchy and centrality measures is still unexplored. In this work, an empirical analysis is conducted to investigate the interplay between hierarchy measures, centrality measures, and the network topology [1]. Three questions are examined. (1) Do hierarchy and centrality measures provide similar information? (2) How does the network topology affect their relationship? (3) Which are the most orthogonal hierarchy and centrality measures? To answer these questions, 6 centrality and 4 hierarchy influential measures are used to quantify the interactions in 28 real-world networks. In order to answer the first question, correlation and similarity analyses are conducted on all the combinations of hierarchy and centrality measures for each network. Results show that hierarchy and centrality measures behave differently, with a range of correlation from high to low as shown in Fig 1 (a). To answer the second question based on correlation/similarity measures, categorization of the networks using the *k*-means algorithm is performed. Inspection of the macroscopic topological properties of the networks reveals that density and transitivity play a major role. If both are high, hierarchy and centrality are well correlated and pretty similar. On the contrary, if one of them is low, hierarchy and centrality measures are quite dissimilar and uncorrelated. Finally, to answer the third question, the Schulze voting method is used. Networks are voters and the hierarchy and centrality combinations are the candidates. It appears that the combination *k*-core, betweenness is the most orthogonal.



(b)

Network Categories	Density	Transitivity	Assortativity
Category 1: Adjective Noun, Zachary Karate, Les Misérables, World Metal Trade, U.S. Airports, Madrid Train Bombings, Birds, and Mammals	High	High	Negative
Category 2: Physicians, Facebook Politician Pages, Facebook Ego, Insects, U.S. States, AstroPh, GrQc, Adolescent Health, Reptiles, and PGP	Low	High	Positive
Category 3: Retweets Copenhagen, Internet A. Systems, NetSci, Human Protein, E. coli Transcription, Mouse Visual Cortex, Yeast Protein, U.S. Power Grids, EuroRoads, and CS Ph.D.	Low	Low	Negative

Figure 1: (a) Heatmaps of the Spearman's correlation (ρ_s) for the various combinations of hierarchy α_i and centrality β_j measures of 6 real-world networks. The hierarchy measures are α_c= *k*-core, α_t= *k*-truss, α_l = LRC, and α_t = triangle participation. The centrality measures are β_d = Degree, β_l = Local, β_b = Betweenness, β_c = Current-flow Closeness, β_k = Katz, and β_p = PageRank. (b) Categories of networks with their aggregated topological characteristics.

References

[1] S. Rajeh, M. Savonnet, E. Leclercq and H. Cherifi, "Interplay Between Hierarchy and Centrality in Complex Networks," in IEEE Access, vol. 8, pp. 129717-129742, 2020, doi: 10.1109/ACCESS.2020.3009525.