Assessing the Relationship Between Centrality and Hierarchy in Complex Networks

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1 Introduction

The topological heterogeneity of real-world networks induces nodes with distinctive roles. Identifying influential nodes within these networks enables us to point out key spreaders for marketing campaigns, finding essential proteins, detecting financial risks, inhibiting diseases and many more [1]. Centrality measures are one of the main ways of assessing a node's importance. These measures are mainly based on the connections of the nodes and the dynamics of the network [2]. Another way of assessing a node's importance is through hierarchy. Hierarchical structure is pervasive and natural among real-world networks [3]. The hierarchical decomposition of networks using hierarchy measures results in nodes that exist at the core of the network. Even though several studies have investigated the relationship between centrality measures [4] [5], no past work was conducted on hierarchy and centrality. As both aim to identify influential nodes, our work [6] looks up on the relationship between hierarchy and centrality, and their association to network's topology. Centrality measures are chosen to incorporate information from the neighborhood (Degree/Local), from the flow of resources (Betweenness/Current-flow Closeness), and from an iterative refinement of the network structure (Katz/PageRank). Hierarchy measures are based on nestedness of the network (k-core/k-truss), flow of resources (LRC), and a mix between nestedness and flow (triangle participation). Hierarchy and centrality measures are calculated on the nodes of 28 diverse real-world networks. Three main questions are investigated to be able to uncover the relationship between centrality and hierarchy from different perspectives. The first one considers if hierarchy and centrality convey the same information. The second one investigates if network topology has an effect on this relationship. Finally, the third one is for examining the orthogonality among centrality and hierarchy.

2 Results

To answer the first question, correlation measures (Pearson, Spearman, and Kendall-Tau) are calculated among the 24 possible hierarchy α_i and centrality β_j measures for each network. Similarity measures are also considered (Jaccard and RBO). Figure 1 reports the Spearman's correlation results of 6 real-world networks exhibiting 6 different yet typical behaviors among the networks. From the first extreme trend, depicted in the CS Ph.D. network, there's no correlation between hierarchy and centrality measures. The second trend, depicted E. coli network, shows only k-core and LRC being

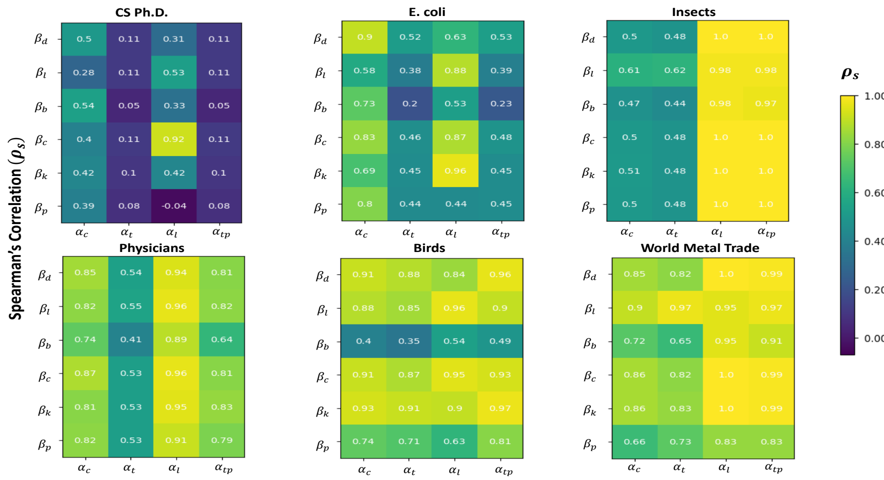

Fig. 1. Heatmaps of the Spearman correlation for the various combinations of hierarchy α_i and centrality β_j measures of 6 real-world networks. The hierarchy measures are α_c = k-core, α_t = k-truss, α_l = LRC, and α_{tp} = triangle participation. The centrality measures are β_d = Degree, β_l = Local, β_b = Betweenness, β_c = Current-flow Closeness, β_k = Katz, and β_p = PageRank.

correlated with centrality measures. The third is having the nested hierarchy (k-core and k-truss) on low correlation, depicted in the Insects network. The fourth trend depicted in the Physicians network shows k-truss being the low correlated hierarchy measure. The fifth case is having all hierarchy measures correlated with all centrality measures except for betweenness. The sixth and final extreme trend, depicted in World Metal Trade, shows high correlation among all hierarchy and centrality measures. These behaviors illustrate variability between hierarchy and centrality among the real-world networks. Nevertheless, 6 trends from the 28 networks can be extracted, showing that underlying interactions are taking place.

Taking the correlation and similarity values a step further, the second set of experiments applies thresholding (μ >0.7) to distinguish between meaningful and non-meaningful values. Networks are then ranked based on the ratio of meaningful correlation (and similarity) values. Table 1 shows the groupings of networks the networks after ranking them based on their meaningful correlation proportion, alongside their main topological characteristics (density, transitivity, and assortativity). A clear association between hierarchy, centrality, and network topology can be seen. The first group (having high fraction of meaningful correlation proportion) is characterized by having high density and transitivity, alongside negative assortativity. The second group (having medium fraction of meaningful correlation proportion) is characterized by low density, high transitivity, and positive assortativity. The third group (having low fraction of meaningful correlation proportion) is characterized by having low density and transitivity with negative assortativity. To prove consistency, k-means was also conducted on the correlation and similarity values. Agnostic to any threshold, k-means showed sim-

Table 1. Real-world networks grouped according to their meaningful correlation proportion. The basic topological characteristics are: ν is the density, ζ is the transitivity, and $k_{nn}(k)$ is the assortativity. Two states can be given to the density and transitivity, either high denoted as H or low denoted as L. Two states can be given for assortativity, either positive denoted as P or negative denoted as N.

Network Groups	ν	ζ	$k_{nn}(k)$
Group 1: Adjective Noun, Zachary Karate, Les Misérables, World Metal Trade, U.S. Airports, Madrid Train Bombings, Birds, and Mammals	H	H	N
Group 2: Physicians, Facebook Politician Pages, Facebook Ego, Insects, U.S. States, AstroPh, GrQc, Adolescent Health, Reptiles, and PGP	L	H	P
Group 3: Retweets Copenhagen, Internet A. Systems, NetSci, Human Protein, E. coli Transcription, Mouse Visual Cortex, Yeast Protein, U.S. Power Grids, EuroRoads, and CS Ph.D.	L	L	N

ilar clusters among the networks. Hence, density and transitivity, and to a lesser extent assortativity, play a major role in the redundancy of information among hierarchy and centrality measures. High density and high transitivity induce high correlation among hierarchy and similarity. On the other hand, having low density and/or low transitivity provides uniqueness of the information extracted by hierarchy and centrality.

Finally, the Schulze method was applied to further investigate the correlation and similarity values between hierarchy and centrality. The nested hierarchy measures (k-core and k-truss) happen to be the most orthogonal to betweenness centrality. Contrary, the most correlated and similar combination is between LRC and current-flow closeness.

Summary. In this work, we have investigated the relationship between hierarchy and centrality. It is found that even though both share the same goal, they are indeed different. The extent of difference or redundancy among them is directly affected by network topology. Specifically, density and transitivity pose as the main building blocks affecting the interplay between hierarchy and centrality.

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