Investigating the Relationship Between Community-aware and Classical Centrality Measures

***S. Rajeh*[1], *M. Savonnet*[1], *E. Leclercq*[1], *H. Cherifi*[1]. [1]*Laboratoire D'Informatique de Bourgogne, University of Burgundy, 21000 - Dijon, France. E-mail: stephany.rajeh@u-bourgogne.fr.*

Community-aware centrality is an emerging research area in network science. Its goal is to exploit the community structure to design effective measures that allow to identify influential nodes in real-world network applications. Although several works have investigated the relationship between various classical centrality measures, the relationship between classical and community-aware centrality measures is still unexplored. This raises two main questions. (1) How community-aware centrality measures compare to classical centrality measures? (2) What is the influence of the various macroscopic and mesoscopic topological properties of the network in the interplay between classical and community-aware centrality measures? In a preliminary work focusing on real-world social networks [1], we showed that community-aware centrality measures do not present a unified behavior. This study extends this work into two directions. First, synthetic data with controlled community structure properties (mixing parameter μ, degree, and community size power-law exponents (γ, θ)) are used to get a better understanding of their relative influence in the interactions between those two types of centrality measures. Kendall's Tau correlation is computed for all possible combinations between 10 classical and 9 community-aware centrality measures on a set of LFR synthetic networks. Results reported in figure 1 (left) show that the behavior of community-aware centrality measures is indeed different from classical centrality measures. Furthermore, the main parameter governing the observed differences is the network community structure strength. In networks with a strong community structure, community-aware centrality measures that rely on inter-community links (local) exhibit low correlation with classical centrality measures. This is not the case for those based on intra-community links (global), which show high correlation. Results are more mixed with community-aware measures based on both intra and inter community links (mixed). The behavior is reversed for networks with weak community structure. Second, a set of 50 real-world networks originating from diverse fields is used in order to link this behavior to the network topological properties. Linear regression is performed considering 6 macroscopic (Density, Transitivity, Assortativity, Average Distance, Diameter, Efficiency) and 9 mesoscopic topological features (Mixing parameter μ, Modularity, Embeddedness, Hub Dominance, Internal Density, Internal Distance, etc.). These are considered as the dependent variables. The independent variables are based on the correlation values obtained from the previous experiment. Results show that transitivity and the mixing parameter are the most significant features ($p < 0.01$). As transitivity increases, correlation between local community-aware and classical centrality measures grows. Furthermore, as the community structure gets weaker, the correlation decreases. The opposite behavior is observed for global community-aware centrality measures. Indeed, as transitivity increases, correlation decreases. Additionally, the correlation decreases when the community structure gets stronger.

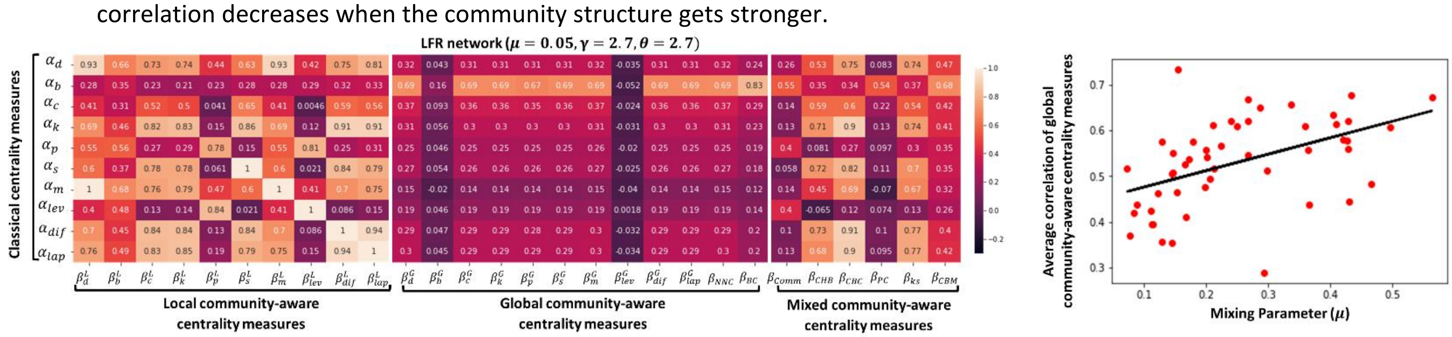

Figure 1 – Left figure: Kendall's Tau correlation for all possible combinations between classical (x-axis) and community-aware (y-axis) centrality measures. The classical centrality measures are: α_d = Degree, α_b = Betweenness, α_c = Closeness, α_k = Katz, α_p = PageRank, α_s = Subgraph, α_m = Maximum Neighborhood Component, α_{lev} = Leverage, α_{dif} = Diffusion, α_{lap} = Laplacian. The local community-aware centrality measures are: (β_d^L, β_b^L, β_c^L, β_k^L, β_p^L, β_s^L, β_m^L, β_{lev}^L, β_{dif}^L, β_{lap}^L) = the local component of the classical centrality measures based on modular centrality. The global community-aware centrality measures are: (β_d^G, β_b^G, β_c^G, β_k^G, β_p^G, β_s^G, β_m^G, β_{lev}^G, β_{dif}^G, β_{lap}^G) = the global component of the classical centrality measures based on modular centrality, β_{NNC} = Number of Neighboring Communities centrality, β_{BC} = Bridging centrality. The mixed community-aware centrality measures are: β_{Comm} = Comm centrality, β_{CHB} = Community Hub-Bridge centrality, β_{CBC} = Community-based centrality, β_{PC} = Participation Coefficient, β_{ks} = K-shell with Community centrality, β_{CBM} = Community-based Mediator centrality. Right figure: Relationship between the mean correlation values of global community-aware centrality measures and the mixing parameter.

[1] Rajeh, Stephany, et al. "Investigating Centrality Measures in Social Networks with Community Structure." International Conference on Complex Networks and Their Applications. Springer, Cham, 2020.